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A GENERAL SOLUTION OF PROBLEM 292.

BY CHAS. H. KUMMELL, U. S. LAKE SURVEY, DETROIT, MICH.

Two surveyors measure a plane field of n sides, one measuring the courses (1.2), (2.3), ... $(n—1.n)$, $(n.1)$ with a chain, and the other measuring the bearings $z_{1.2}$, $z_{2.3}$, ... $z_{n-1.n}$, $z_{n.1}$ with a theodolite. From former experience it is known that the first is liable to a probable error of c inches per chain and the other to w'' per angle. Required, normal equations for determining the most probable corrections to the measured quantities.

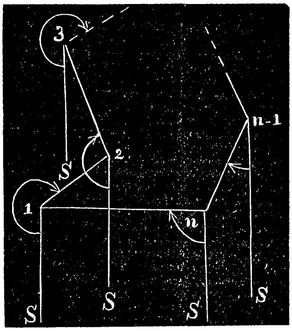
Denoting the angles by (1), (2), ... $(n—1)$, (n) and corrections to observed quantities by prefixing δ we have

$$z_{1,2}+\delta z_{1.2} = \pi+z_{n.1}+\delta z_{n.1}-(1)-\delta(1), \quad (1_1)$$

$$z_{2.3}+\delta z_{2.3} = 2\pi+z_{n,1}+\delta z_{n.1}-(1)-\delta(1) -(2)-\delta(2), \quad (1_2)$$

. . . .

$$z_{n-1.n}+\delta z_{n-1.n} = (n-1)\pi+z_{n.1}+\delta z_{n.1}-(1)- \delta(1)-(2)-\delta(2)-\ldots-(n-1)-\delta(n-1), \quad (1_{n-1})$$



$$2\pi+z_{n.1}+\delta z_{n,1} = n\pi+z_{n,1}+\delta z_{n.1}-(1)-\delta(1)-(2)-\delta(2)-\ldots-(n)-\delta(n). \quad (1_n)$$

Placing in (1_n) $(n-2)\pi-(1)-(2)-\ldots-(n) = E$, we have

$$\delta(1)+\delta(2)+\ldots+\delta(n)-E=0 \qquad (I)$$

We have the sum of the Northings and Southings

$$[(1.2)+\delta(1.2)]\cos(z_{1.2}+\delta z_{1.2})+[(2.3)+\delta(2.3)]\cos(z_{2.3}+\delta z_{2.3})+\ldots +[(n.1)+\delta(n.1)]\cos(z_{n.1}+\delta z_{n.1}) = 0.$$

This condition is true whatever $z_{n.1}$ is. The same is the case with the condition of Eastings and Westings. There are therefore no conditions to determine its correction, that is we cannot orient the figure without another determination of absolute azimuth.

We have, regarding (1)

$$[(1.2)+\delta(1.2)]\cos[z_{1.2}+\delta z_{n.1}-(1)]+[(2.3)+\delta(2.3)]\cos[z_{2.3}+\delta z_{n\,1}-\delta(1) -\delta(2)]+\ldots+[(n-1.n)+\delta(n-1.n)]\cos[z_{n-1.n}+\delta z_{n.1}-\delta(1)-\delta(2)-\ldots -\delta(n-1)]+[(n.1)+\delta(n.1)]\cos(z_{n.1}+\delta z_{n.1}) = \cos z_{1.2}\,\delta(1.2)+\cos z_{2.3}\,\delta(2.3) +\ldots+\cos z_{n-1.n}\,\delta(n-1.n)+\cos z_{n.1}\,\delta(n.1)+W_{1.n}\sin 1''\delta(1)+W_{2,n}\sin 1''\delta(2) +\ldots+W_{n-1.n}\sin 1''\delta(n-1)-W_{1.n+1}\sin 1''\delta z_{n.1}+S_{1.n+1} = 0, \qquad (II)$$

where $W_{r.n}$ denotes the projection of the diagonal $(r.n)$ on the East and West axis and $S_{r.n}$ that on the North and South axis.

From the condition that the corrected eastings and westings must balance we have similarly

$$\sin z_{1.2}\delta(1.2)+\sin z_{2.3}\delta(2.3)+\ldots+\sin z_{n-1.n}\,\delta(n-1.n)+\sin z_{n.1}\,\delta(n.1)$$
$$-S_{1.n}\sin 1''\delta(1)-S_{2.n}\sin 1''\delta(2)-\ldots-S_{n-1.n}\sin 1''\delta(n-1)+S_{1.n+1}\sin 1''$$
$$\times\delta z_{n.1}+W_{1.n+1}=0. \qquad \text{(III)}$$

Because the coefficients of $\delta z_{n.1}$, viz., $W_{1.n+1}$ and $S_{1.n+1}$ must be nearly $=0$, these terms may be omitted, being small quantities of the second order.

If (1.2) is given in inches there are $\frac{1}{792}(1.2)$ or (putting $\frac{1}{792}=m$) $m(1.2)$ chains in it; it is therefore liable to a probable error $=c\surd[m(1.2)]$ and has the absolute weight $=792\div(1.2)c^2$. Similarly the other sides.

The weights of the angles (1), (2), ... (n) are, on the same scale $=1\div w^2$.

We have then the following minimum equation:

$$\frac{792}{c^2}\left\{\frac{\delta(1.2)^2}{(1.2)}+\frac{\delta(2.3)^2}{(2.3)}+\ldots+\frac{\delta(n.1)^2}{(n.1)}\right\}+\frac{1}{w^2}\left\{\delta(1)^2+\delta(2)^2+\ldots\delta(n)^2\right\}$$
$$-2[\Delta^*(1)+\Delta(2)+\ldots+\Delta(n)-E]C_1$$
$$-2[\cos z_{1.2}\Delta(1.2)+\cos z_{2.3}\Delta(2.3)+\ldots+\cos z_{n-1.n}\Delta(n-1.n)+\cos z_{n.1}\Delta(n.1)$$
$$+W_{1.n}\sin 1''\Delta(1)+W_{2.n}\sin 1''\Delta(2)+\ldots+W_{n-1.n}\sin 1''\Delta(n-1)+S_{1.n+1}]C_2$$
$$-2[\sin z_{1,2}\Delta(1.2)+\sin z_{2.3}\Delta(2.3)+\ldots+\sin z_{n-1.n}\Delta(n-1.n)+\sin z_{n.1}\Delta(n.1)$$
$$-S_{1.n}\sin 1''\Delta(1)-S_{2.n}\sin 1''\Delta(2)-\ldots-S_{n-1.n}\sin 1''\Delta(n-1)+W_{1.n+1}]C_3, \quad (2)$$

where C_1, C_2, C_3 are undetermined factors or correlates.

Taking partial differential coefficients and solving for the corrections we obtain:

$$\Delta(1)=w^2[C_1+W_{1.n}\sin 1''C_2-S_{1.n}\sin 1''C_3], \qquad (3_1)$$
$$\Delta(2)=w^2[C_1+W_{2.n}\sin 1''C_2-S_{2.n}\sin 1''C_3], \qquad (3_2)$$

. . . .

$$\Delta(n-1)=w^2[C_1+W_{n-1.n}\sin 1''C_2-S_{n-1.n}\sin 1''C_3], \qquad (3_{n-1})$$
$$\Delta(n)=w_2[C_1 \qquad], \qquad (3_n)$$
$$\Delta(1.2)=mc^2(1.2)[\cos z_{1.2}\,C_2+\sin z_{1.2}\,C_3], \qquad (4_1)$$
$$\Delta(2.3)=mc^2(2.3)[\cos z_{2.3}\,C_2+\sin z_{2.3}\,C_3], \qquad (4_2)$$

. . . .

$$\Delta(n-1.n)=mc^2(n-1.n)[\cos z_{n-1.n}C_2+\sin z_{n-1.n}C_3], \qquad (4_{n-1})$$
$$\Delta(n.1)=mc^2(n.1)\;[\cos z_{n,1}\;\;C_2+\sin z_{n.1}\;\;C_3], \qquad (4_n)$$

Substituting these expressions into (I), (II) and (III) we obtain the normal equations:

$$0=-E+nw^2C_1+w^2\sin 1''[W_{r.n}]C_2-w^2\sin 1''[S_{r.n}]C_3$$
$$0=S_{1.n+1}+w^2\sin 1''[W_{r.n}]C_1+\{w^2\sin^2 1''[W_{r.n}^2]+mc^2[(r.r+1)\cos^2 z_{r.r+1}]$$
$$\}C_2-\{w^2\sin 1''[W_{r.n}S_{r.n}]-mc^2[(r.r+1)\sin z_{r.r+1}\cos z_{r.r+1}]\}C_3,$$

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*Used to represent δ.

$$0 = W_{1.n+1} - w^2 \sin 1''[S_{r.n}]C_1 - \{ w^2 \sin^2 1''[W_{r.n}S_{r.n}] - mc^2[(r.r+1) \times \sin z_{r.r+1} \cos z_{r.r+1}] \} C_2 + \{ w^2 \sin 1''[S_{r.n}] + mc^2[(r.r+1)\sin^2 z_{r.r+1}] \} C_3. \quad (5)$$

Having solved these equations, formulæ (3) and (4) will give the required corrections to the observed quantities. The weighted sum of squares of these corrections may then be computed, either directly or indirectly by the formula

$$\frac{1}{w^2}\left[\varDelta(r)^2 \right] + \frac{792}{c^2}\left[\frac{\varDelta(r.r+1)^2}{(r.r+1)} \right] = EC_1 - S_{1.n+1}C_2 - W_{1.n+1}C_3. \quad (6)$$

The probable error of weight 1 is then

$$r = 0.6745\,\sqrt{[\tfrac{1}{3}(EC_1 - S_{1.n+1}C_2 - W_{1.n+1}C_3)]}. \quad (7)$$

The published solution of this problem as given by Mr. Adcock at p. 60 is not correct for several reasons.

1. He adjusts the angles independent of their relation to the sides and then uses the resulting angle of this partial adjustment as absolutely correct in the adjustment for the sides.
2. He considers the sides all equally good although I only said that the measurement of the sides is liable to c inches per chain; therefore if for instance the side a contains a feet it is measured by $(a \div 66)$ chains, each of which is liable to c inches probable error. Then the sides a, b, etc., are liable to the probable errors

$$c\sqrt{\frac{a}{66}},\quad c\sqrt{\frac{b}{66}},\ \text{etc., respectively.}$$

These are well recognized principles all over the world.

THE NINE-LINE CONIC.

BY MISS CHRISTINE LADD, BALTIMORE, MARYLAND.

THE properties of the nine-line conic may be obtained directly, or by reciprocation from those of the nine-point circle. I shall make use of the latter method. Some of these properties are well known; those which I suppose to have been unnoticed hitherto I shall indicate by Italics. Large letters will, as usual, represent points and small letters lines. As two large letters, BC, denote a line, so do two small letters, bc, denote a point.

$ABC \equiv$ triangle.

$a\,b\,c \equiv$ trilateral.

$O \equiv$ centre of circumscribed circle.

$o \equiv$ directrix of inscribed conic with arbitrary focus, F.